SCIENCE AND CIVILISATION IN CHINA

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fected by Blaise Pascal two years later; its essential point of superiority was that it carried over the tens automatically by a mechanism of gearing instead of this having to be done by the operator.[a] Naturally in due course slide-rules[b] and simple calculating machines found their way to China. Fig. 68, from Michel (5), shows a Chinese slide-rule of +1660.

One can find occasional references to unfamiliar pieces of mathematical apparatus resembling graduated counting-rods. For example, the *Chieh An Man Pi*[1] (Notes from the Fasting Pavilion)[c] says that

> there was a *chhien hsing pan*[2] (lit. a hand-worked star board)[d] constructed by Ma Huai-Tê[3] (*fl. c.* +1064) at Suchow. A set of it contained 12 pieces of ebony ranging in length from 7 inches downwards. They were graduated in finger-breadths up to 12, and each had also smaller divisions as well. There was also a piece of ivory with its four corners left blank, and 2 inches long. It had the words *pan chih*[4] (half finger), *pan chio*[5] (half angle) and *i chio*[6] (full angle) carved on it. These rulers faced each other reciprocally. They were called *Chou Pei Suan*[7] rulers.[e]

No explanation has come down to us as to the use of these rulers but they would seem to have been for geometrical purposes.

(3) The Abacus

As to the Chinese abacus, which has given rise to a large literature,[f] it will be advisable to begin by describing it briefly.[g] It is called *suan phan*[8] (calculating plate) or *chu suan phan*[9] (ball-plate). It consists today of a rectangular wood frame, the longer sides of which are connected by means of wires forming a series of parallel columns (*wei*,[10] *hang*[11] or *tang*[12]). Each of these has threaded on it seven slightly flattened balls (*chu*[13]), which can be moved to or away from a long transverse bar (*liang*[14] or *chi liang*[15]), dividing the abacus into two unequal parts, so that two balls always remain above, and five below, the separating bar. There are usually twelve wires, but there may be as many as thirty. Each of the balls above the bar is equivalent to the five balls below in

[a] Lilley (1); Taton (1); Baxendall (1).

[b] Misunderstanding may sometimes have been caused by an unfortunate phrase used by Mikami (1), pp. 13, 14. In giving an account of the Han method of finding square roots, he referred to the placing of one ordinary counting-rod at the bottom of the counting-board to signify 1, 10 or its powers. This he called 'borrowing a unit calculator'. Needless to say, this has nothing remotely to do with borrowing a slide-rule from a friend.

[c] Probably part of the *Chieh An Chi*[16] by Chin Kuei[17] (*fl. c.* +1500).

[d] The first of these characters is a rare one. The 'stars' may have been pins hinging the rulers together.

[e] Tr. auct.

[f] From the mass of papers, the following may be selected as valuable for consultation: Goschkevitch (1), Rodet (1), Westphal (1), van Name (1), Knott (1), Kuo Mai-Ying (1), Yen Tun-Chieh (3), Leavens (1), Rohrberg (1), Yoshino (1).

[g] This instrument cannot be judged by the dwindled and degenerate form of it which till recently persisted in European infant schools. It is used in China and Japan in a particularly scientific way, and still holds its own outside the narrow circle of those concerned with higher mathematics.

1 戒菴漫筆 2 搲星板 3 馬懷德 4 半指 5 半角
6 一角 7 周髀算 8 算盤 9 珠算盤 10 位 11 行
12 檔 13 珠 14 梁 15 脊梁 16 戒菴集 17 靳貴

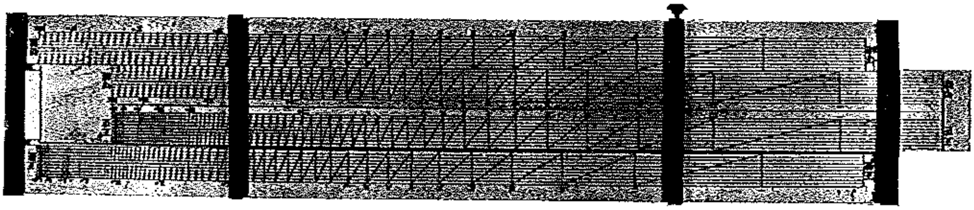


Fig. 68. A Chinese slide-rule of +1660 (photo. Michel).

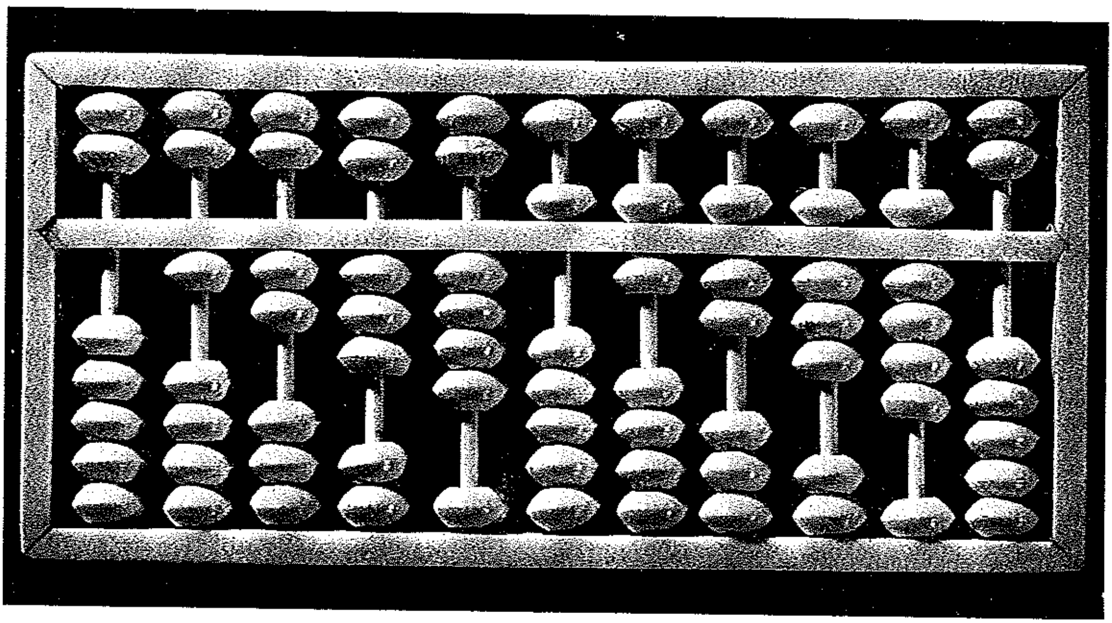


Fig. 69. Chinese abacus (orig. photo.).

the same vertical column, and each of the columns differs by ten, so that each ball in any given column is equivalent to ten similarly placed balls of the column immediately on its right. Exactly what units each column stands for varies at the will of the operator. The use of the abacus may be seen from Fig. 69 where the number 123,456.789 has been set up.[a] It is possible to carry out the first three fundamental operations using only one of the balls in the upper register, but for division it is convenient to be able to indicate on any given column a number higher than 10, so that two upper balls as well as five lower ones are provided, giving a total of 15. The left side of the abacus is considered the front (*chhien*[1]) and the right the back (*hou*[2]); 'advancing' a number (*chin*[3]) means raising it by powers of ten, while 'retiring' it (*thui*[4]) means the opposite. Setting up a number is to '*shang*[5]' it; for cancelling a number the verbs *chhi*,[6] *chhu*[7] or *chhü*[8] are used. The left-hand column of a calculation is the 'head' (*shou*[9] or *thou*[10]), the last or right-hand column is the 'tail' (*wei*[11] or *mo*[12]).[b] The column which is being operated upon is the '*body*' (*shen*[13]) or principal position (*pên wei*[14]). The detailed operations in the use of the abacus, including the extraction of square and cube roots, are described by Knott (1) and by Smith & Mikami (1). Its special advantages, for example, in accounting, even at the present day, are alluded to by Leavens. In order to illustrate the astonishing speed which Chinese and Japanese computers, trained to the use of the instrument from childhood, can attain, it may be mentioned that when in 1946 a competition was staged in Tokyo between an abacus clerk and an American sergeant using an electric calculating machine, the abacus won for speed in all operations except multiplication, and also in making fewer mistakes (Kojima, 1). Of course, with all simple mechanical aids to computation, the intermediate stages leave no trace, and checking is therefore difficult.

We have now to examine the history of the instrument. The fact that there is no complete description of the abacus in its modern form before the *Suan Fa Thung Tsung* of Chhêng Ta-Wei (+1593)[c] (see Fig. 70) has led many, including Mei Wên-Ting, to conclude that it did not become known in China until the end of the +15th century. Yet the earliest illustration of it is in the *Hsin Pien Tui Hsiang Ssu Yen*[15] of +1436 (Goodrich, 5).[d] Besides, no one has noticed that the *Lu Thang Shih Hua*[16] (Foothill Hall Essays) of +1513, written by Li Tung-Yang,[17] describes the abacus clearly as the

[a] It will be seen that only those balls count in the calculation which have been placed in contact with the transverse bar.

[b] These terms occur as early as the +3rd century in the biography (*San Kuo Chih*, ch. 63, pp. 4*a* ff.) of a famous mathematician and diviner of the Wu State, Chao Ta[18] (*fl.* +225 to +245). He had a method of computation called *thou chhêng wei chhu*[19] (multiplying at the head and dividing at the tail), which was vainly sought after by contemporary officials such as Kungsun Thêng.[20] However, the interpretation of this remains uncertain and we shall return to the matter in Sect. 26*i*.

[c] There were many descriptions after that time; references to books especially devoted to abacus calculations in the 17th and 18th centuries will be found in Wieger (3), p. 264, and Wylie (1), p. 103.

[d] This is of much interest in itself, as the oldest illustrated children's primer in the world, more than two centuries earlier than the *Orbis Sensualium Pictus* of Comenius (+1658).

1 前 2 後 3 進 4 退 5 上 6 起 7 除
8 去 9 首 10 頭 11 尾 12 末 13 身 14 本位
15 新篇對象四言 16 麓堂詩話 17 李東陽 18 趙達
19 頭乘尾除 20 公孫滕

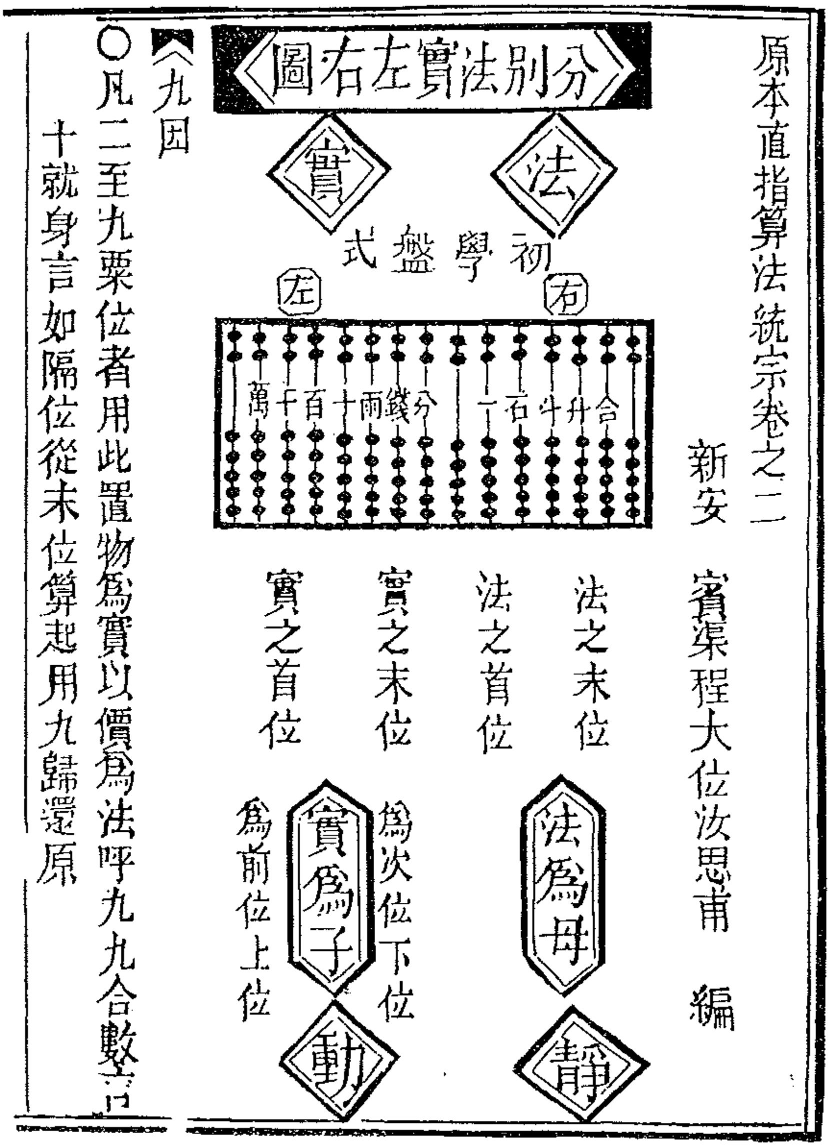


Fig. 70. An early printed picture of the abacus, from the *Suan Fa Thung Tsung*, +1593.

'moving-ball plate' (*chu chih tsou phan*[1]) which was operated according to rules and standard methods.[a] But here enters in the difficult problem of the *Shu Shu Chi I* (Memoir on some Traditions of Mathematical Art), which is attributed to Hsü Yo at the end of the Later Han (about +190), but which may possibly have been actually written by its commentator, Chen Luan (*c.* +570). In either case, it is much the earliest work which speaks of 'ball arithmetic'. According to Hsü Yo, his teacher

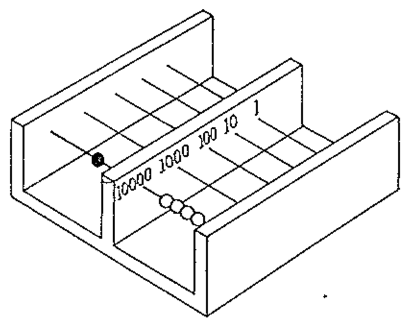
[a] Pp. 3*a*, 4*b*.

[1] 珠之走盤

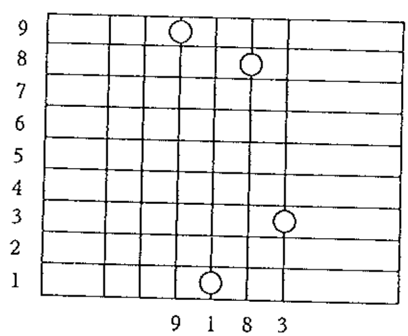
Liu Hui-Chi [1] visited a Taoist adept, Thien Mu hsien-sêng,[2] who explained to him fourteen old methods of calculation, one of which was actually called ball-arithmetic (*chu suan*[3]). As this has not before been englished, it is given here:

(Text) The ball-arithmetic (method) holds and threads together (*khung tai*[4]) the Four Seasons, and fixes the Three Powers (heaven, earth and man) like the warp and weft of a fabric.

(Commentary) A board is carved with three horizontal divisions, the upper one and the lower one for suspending the travelling balls (*yu chu*[5]), and the middle one for fixing the digit[a] (*ting suan wei*[6]). Each digit (column) has five balls. The colour of the ball in the upper division is different from the colour of the four in the lower ones. The upper one corresponds to five units, and each of the four lower balls corresponds to one unit. Because of the way in which the four balls are led (to and fro) it is called 'holding and threading together the Four Seasons'. Because there are three divisions among which the balls travel, so it is called 'fixing the Three Powers like the warp and weft of a fabric'.[b]



It must be admitted that this is a remarkably clear description of some kind of abacus, obviously one in which the total available number of units in each column was 9. It could be pictured as a trough-and-ball instrument if it were not for the word *tai*[7] (belt, ribbon) which has unmistakably the sense of a wire.



The commentary on three of the other methods also mentions balls. Mikami (*1*) has made an effort to elucidate what they were from their obscure descriptions. In the *Thai I*[8] (Great Unity) method, the text speaks of something coming and going along nine *tao*,[9] ways or lines (perhaps troughs?). The commentary says that there was one ball (hence the name of the method) for each vertical column, which was divided into nine horizontal divisions; hence by moving the balls up and down, any number which it was desired to retain could be set up. This method brings out clearly the way in which coordinate geometry was latent in the abacus system, the graduation into powers of ten forming the x-axis, and the graduation of numbers less than ten forming the y-axis. If the balls could ever have been persuaded, even in thought, to move along continuous curves, what a Cartesian world of graphs would have opened out!

a I.e. whether tens, hundreds, etc. This was a good idea. The modern abacus is always unmarked.
b Tr. auct.

1 劉會稽 2 天目先生 3 珠算 4 控帶 5 游珠
6 定算位 7 帶 8 太一 9 九道

Another method, the *Liang I*[1] (Heaven and Earth method), used balls of two different colours, yellow and blue, one relating to a *y*-axis on the left and the other to a *y*-axis on the right. Numbers were set up in the same manner as the *Thai I* method. This bears an astonishing resemblance to the practice in modern graphic representations of curves where points of different types relate to different superimposed axis graduations. In the third of these methods, balls of three different colours were used, and there were only three horizontal positions. In this way again any desired number could be set up. This was the *San Tshai*[2] (Three Powers)[a] method. On the whole, these systems, even if as late as the +6th century, show an interesting appreciation of coordinate relationships.[b]

A reference to the abacus may perhaps be contained in the biography of the metallurgist Chhiwu Huai-Wên[3] (*fl.* +550 to +570) of the Northern Chhi dynasty.[c]

> Many say that once in the College at Chinyang, when a Jun-Jun (i.e. Hun) guest was present, a foreign Buddhist monk, pointing to (Chhiwu) Huai-Wên, said to him, 'Here's a man who has strange mathematical arts!' And indicating a jujube tree in the courtyard he asked Huai-Wên to use the calculator (*suan tzu*[4]) and say how many dates there were on the tree. The trial was made, and Huai-Wên stated not only the total number but how many were ripe, unripe and partly ripe. When the dates were counted and tested they were only one short, but the mathematician said, 'It can't be wrong, just shake the tree again.' And sure enough, one more fell down.[d]

Here counting-rods may be implied, though the 'calculator' sounds like an abacus.

After the *Shu Shu Chi I* there is a long silence about the instrument. However, the bibliography of mathematical books which Chhêng Ta-Wei annexed to his work of +1593 states that between +1078 and +1162 there existed four books which, to judge from their titles, concerned the abacus. These were the *Phan Chu Chi*[5] (Record of the Plate and the Balls), the *Tsou Phan Chi*[6] (Record of the Moving Plate), the *Thung Wei Chi*[7] (Record of the Communicating Small Objects), and the *Thung Chi Chi*[8] (Record of the Communicating Machine).[e] None of them has come down to us, and it is doubtful whether Chhêng Ta-Wei himself had seen them. Vissière was

[a] Not mathematical powers, but Heaven, Earth and Man.

[b] Cf. my remark above (Vol. 1, p. 34) concerning rhyme-tables. Of course the coordinate concepts in Apollonius' conic sections (−3rd century), involving continuous variables, more fundamentally foreshadow analytic geometry.

[c] *Pei Shih*, ch. 89, p. 20*a*.

[d] Tr. auct. For an Indian parallel to this story see the legend of Nala and Rituparna in the *Mahābhārata* (Ray tr., vol. 3 (Vana Parva), ch. 72, p. 215). Chhiwu Huai-Wên's great importance in the development of iron and steel technology will be seen in Sect. 30*d*.

[e] All these books were said by Chhêng Ta-Wei to have been mentioned by Yang Hui in his +1275 book on magic squares, already referred to, but they are not in the text of that as we now have it. None of them is mentioned in the HY Index of titles in the official bibliographies, with the exception of a *Thung Wei Tzu*[9] (if it is the same book) of the Thang, which included a *Shih Wu Chih*,[10] 'Record of the Ten Things' (Balls?). The Sung bibliography also has a *Thung Wei Miao Chüeh*[11] (Mysterious Secret of the Communicating Small Objects), anonymous but possibly relevant. See *Suan Fa Thung Tsung*, ch. 12, p. 20*b*.

[1] 兩儀 [2] 三才 [3] 綦毋懷文 [4] 筭子 [5] 盤珠集
[6] 走盤集 [7] 通微集 [8] 通機集 [9] 通微子 [10] 十物志
[11] 通微妙訣

sceptical about their relevance, but the impression which their titles give of having concerned the abacus is really rather strong.

Mei Wên-Ting was of the opinion that the use of the abacus had first become general about the time of the mathematician Wu Hsin-Min[1] (*fl. c.* +1450).[a] He also considered that the computations for the Ta Thung Li calendar of +1384, which had been prepared under the leadership of Yuan Thung[2] with the help of Kuo Po-Yü,[3] a descendant of the great Kuo Shou-Ching, had been made with the abacus.[b] But, as Li Nien has pointed out,[c] Mei overlooked the fact that Chhêng Ta-Wei quoted fragments from the now lost book of Hsieh Chha-Wei,[4] who was probably a contemporary of Shen Kua and would have flourished in the last quarter of the +11th century, which clearly refer to the abacus and even use the expression *chi-liang*[5] (backbone) for the dividing bar. Yen Tun-Chieh (*3*) has added a poem of +1279 by Liu Yin[6] which mentions the term *suan phan*.

A few words must now be said about the comparative history of the abacus in other civilisations. The etymology of the Latin word is obscure, but it probably originated from a Semitic word *abq* (dust), recalling the fact that the earliest precursor of the abacus was probably a dust or sand tray. The next step was to have a surface ruled in lines on which pebbles (*calculi*) or counters could be placed, and there is some evidence that this was first used in ancient India,[d] though Herodotus says that the Egyptians had been accustomed to reckon with pebbles.[e] Concrete evidence is afforded by the famous Salamis abacus, with its parallel lines ruled in marble;[f] unfortunately it cannot be dated. In Latin literature there are numerous references to abaci having ruled lines and used with pebbles,[g] and in modern times several examples of metal plates with knobs running in slots have found their way into museums; these however, like the Salamis marble, are of quite uncertain date. The position can be summed up by saying that if these specimens are placed as late as the +3rd or +4th century (which would be quite reasonable), and if the dating of Hsü Yo's text is accepted as of the end of the +2nd century, then the Chinese practice would slightly antedate the European.[h] Since so

[a] His *Chiu Chang Pi Lei Suan Fa*[7] (Comparative Study of the 'Nine Chapters') has long been lost, but it is known to have mentioned the abacus.

[b] Chhien Ta-Hsin[8] (*Shih Chia Chai Yang Hsin Lu*,[9] ch. 17, p. 3*b*) was able to push this back a few decades by noting certain references in the *Cho Kêng Lu*[10] of Thao Tsung-I[11] of +1366.

[c] (*2*), 1st ed. p. 171, 2nd ed. p. 162, (*4*), vol. 3, p. 37, (*21*), vol. 4, p. 21. The reference is to *Suan Fa Thung Tsung*, ch. 1, p. 2*b*. Cf. Mikami (1), p. 61.

[d] Kaye (2).

[e] Smith (1), vol. 2, p. 160.

[f] Rangabé (1); Kubitschek (1).

[g] Smith (1), vol. 2, p. 165.

[h] What is the relation of the abacus to the rosary used for religious or magical purposes? There seems to be general agreement that India was its original home, and that the *japa-mala* (muttering chaplet) was used by Buddhists there in the +1st century. Islam had it by the +9th and Christendom by the +11th; one of the first Chinese references concerns a court eunuch of the +8th. The development and spread of the rosary seems to go parallel to that of the abacus; may not the chaplet have given rise to the instrument? Cf. Kirfel (1).

[1] 吳信民 [2] 元統 [3] 郭伯玉 [4] 謝察微 [5] 脊梁 [6] 劉因
[7] 九章比類算法 [8] 錢大昕 [9] 十駕齋養新錄 [10] 輟耕錄
[11] 陶宗儀

much uncertainty is involved, however, it is not possible to consider the question as in any way settled.[a]

The abacus did not become common in Europe until the +11th and +12th centuries.[b] There were several treatises on calculation *super lineas et per projectiles*, such as that of Hermann the Lame about +1050, which would correspond to such books as the *Phan Chu Chi*. A persistent European medieval tradition derived the abacus from the Arabs, but such a transmission is not established, and the origin of the Arabic abacus itself, which had ten balls on each wire, and no centre bar, is much in doubt.[c] The boards with lines were known as counting-tables, and the pebbles used were called jettons or casting-counters.[d] The counting-table was also, because of its chessboard appearance, an 'exchequer'—hence the modern term Chancellor of the Exchequer. It has been suggested that one form of it in Europe, that of the Banque des Argentiers, had been a direct importation of the Chinese *suan phan*,[1] and this may well be true, irrespective of other origins or transmissions, since it would have been something in which the +13th-century commercial travellers in Asia, such as Marco Polo, would naturally have taken an interest. The Russians, who maintained the use of the abacus down to very recent times, have sometimes attributed a Chinese origin to it.[e] In its specifically Chinese form it attracted much interest in 17th- and 18th-century Europe, and was described by Martini (+1658), Spizel (+1660), de la Loubère (+1691) and by two authors in the *Philosophical Transactions of the Royal Society*, Gamaliel Smethurst in +1749 and Robert Hooke himself in +1686.

In disposing of this subject we may notice a few final points. It seems unlikely that the character *suan*[2] originated from a graphic representation of an abacus (cf. p. 4 above). The argument of de Lacouperie that if the *suan phan*[1] were really pre-Han it would have been represented by a single character instead of a compound expression is still not without weight. Knott's suggestion that it had a foreign origin because the numbers are set up from left to right on it, not from right to left, loses all plausibility when we remember not only that each individual Chinese character is written from left to right, but also that the ancient counting-board was worked in that way. Perhaps the best provisional conclusion is that of Sarton,[f] that independent inventions took place. It will not be the last time that we shall have to fall back on this suspended judgment.

[a] It is clear that the whole treatment of this subject by D. E. Smith is out of date and requires reconsideration. He repeats (vol. 2, p. 169) the error of de Lacouperie (2) which arose from the misprint in the *Thai-Phing Yü Lan* (see p. 71 above). One of the latest histories of mathematics, Becker & Hofmann (1953) places the Chinese abacus not before the +12th century, thus neglecting Chen Luan, to say nothing of Hsü Yo (p. 133). What may perhaps be a representation of an abacus in the reliefs of the Han tombs at I-nan (*c.* +193), contemporary with Hsü Yo himself, has just been discovered.

[b] Sarton (1), vol. 1, p. 756. But Chen Luan's contemporary, Boethius, knew and used it.

[c] Gandz (2).

[d] Barnard (1).

[e] Not, however, Spassky (1), in the most recent study of the *schioty*.

[f] (1), vol. 1, p. 757.

[1] 算盤 [2] 算